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Determination of QCD condensates without hadronic spectra

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Abstract

The bounds on the values of gluon, four-quark and quark-gluon condensates are derived from the requirement of consistency of the sum rules for various correlators of the hybrid current $a_\mu = g\bar{d}\gamma_\rho\tilde{G}_{\rho\mu}u$. The upper bound for the gluon condensate is found to be less than twice its standard value while for the four-quark condensates the violation of factorization by a factor 3-4 is allowed. The value of the quark-gluon condensate is given as a function of $\langle\frac{\alpha_s}{\pi}G^2\rangle$ and $\langle(\bar{q}\Gamma q)^2\rangle$ which for the standard values yields $m_0^2 = 0.63 \text{ GeV}^2$. Our procedure of simultaneous solution of a number of sum rules allows the quantitative error analysis and is appropriate for condensate determination also in other cases.

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Introduction

QCD vacuum condensates, the vacuum expectation values of local operators, introduced in ref.[1] as essential ingredients of the QCD sum rules (SR) method, are the fundamental parameters of theory. They parametrize the properties of QCD vacuum and therefore have relation to almost all aspects of the hadron physics.

Since the introduction of QCD condensates, considerable effort has been devoted to determination of their values, which however up to now has given a little effect. The existing theoretical schemes provide only qualitative estimates, while semiphenomenological analyses on the basis of QCD SR approach have resulted in unacceptably wide ranges of possible values, even for the basic condensates of lowest dimensions.

The most commonly used values of QCD condensates are the so called "standard values" (SV) of ref.[1]. For the gluon condensate this is

$$\left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \right\rangle_{st} = (330 \text{ MeV})^4 \quad (1)$$

as derived from charmonium phenomenology, while for the four-quark condensate - the "factorized" value

$$\langle (\bar{q}\Gamma q)^2 \rangle_{fact} = \frac{1}{N(\Gamma)} < \bar{q}q >^2 \quad (2)$$

with the quark condensate given by $< \sqrt{\alpha} \bar{q}q > = (-240 \text{ MeV})^3$, and $N(\Gamma)$ - a normalization factor depending on the matrix Γ .

The mixed quark gluon condensate has been estimated from the baryon sum rules [2] to be

$$\left\langle ig\bar{q}\sigma_{\mu\nu}G_{\mu\nu}^a \frac{t^a}{2}q \right\rangle = m_0^2 < \bar{q}q > \quad (3)$$

with $m_0^2 = 0.8 \text{ GeV}^2$.

In a great number of articles the different types of sum rules (Borel SR [1], FESR[3], Gauss SR [4] etc.) have been applied for extraction of the actual condensate values from experimental data [5, 6] (here we quote only a few). Most of them use the measured hadronic spectra from e^+e^- annihilation or τ decay processes. These studies however have not led to a consistent picture. The resulting values of condensates considerably differ from each other, most of them exceed the SV several times. For the gluon condensate the corresponding factor varies in the range (1-6) while for the four-quark condensate it is up to an order of magnitude. As for the parameter m_0^2 , the estimates ranging from 0.2 to 1.1 GeV^2 can be found in the literature.

On the other hand there are still arguments against such strong deviations of the vacuum condensates from their SVs. So in ref.[7] the restrictions on the value of gluon condensate have been obtained from the requirement of consistency of different SR for the correlators of a hybrid current $a_\mu = g\bar{d}\gamma_\rho\tilde{G}_{\rho\mu}u$. The two point functions $< 0|T(a_\mu, a_\nu)|0 >$ and $< 0|T(\bar{u}\gamma_\mu\gamma_5 d, a_\nu)|0 >$ have been considered and the gluon condensate has been bounded by the requirement that these different sum rules yield the same result for the pion matrix element $< 0|a_\mu|\pi(p) > = -if_\pi\delta^2 p_\mu$.

Using the factorization assumption for the four-quark condensate, the authors have concluded that $\langle \frac{\alpha}{\pi} G_{\mu\nu}^2 \rangle$ is forbidden to exceed its standard value by more than 40%. The value of the quark-gluon condensate has been calculated in [8] by means of the sum rules for the correlator of a_μ with the pseudoscalar current. The authors, using the SV for the four-quark and the gluon condensates together with $\delta^2 = 0.2$ from ref.[7], have arrived at the value of eq.(3).

The mentioned uncertainties of the condensate values lead to considerable ambiguities in the predictions of QCD SR as well as in various other applications. They also arise doubts in the consistency of description of the vacuum properties by means of local condensates. However considering the situation with condensate determination one has to remember that QCD SR is a semiquantitative method and without well defined error analysis the results have quite a limited value. Unfortunately not all of the attempts of condensate determination meet this requirements. The intrinsic uncertainties of the method related to the truncation of power series and to the imperfection of the model spectra are not even parametrized in a fully satisfactory way. Hence different types of SR, influenced by these uncertainties in different ways, produce the results which seem to be hardly compatible with each other. Presently the systematic study of different sum rules with clear statements about related errors is highly desirable; This will allow either to constrain the values of the basic condensates to some reasonable ranges or to show the limits of consistency of the whole approach.

In the present paper we perform the simultaneous analysis of the sum rules for different correlators of the current a_μ in order to impose bounds on the values of the gluon quark-gluon and the four-quark condensates. We are motivated by several reasons. First of all let us note that in ref.[7] the gluon condensate has been fixed by assuming the SV for the four-quark condensates. On the other hand we have quite a reliable estimate based on the τ decay data and low energy theorems of current algebra, showing that for the definite four-quark condensate the factorization approximation is violated by a factor of (2.5-3) [6]. So it is desirable to find bounds on the above parameters without any ad hoc. assumptiontions. It is also interesting to find the allowed region for δ^2 which is an important parameter determining many dynamical characteristics of pion. Our motivation is also to present the framework which allows the quantitative error analysis and is more adequate for a systematic study of a number of different sum rules.

Our suggestion briefly is the following. Instead of the standard procedure we propose to consider the sum rules at different values of the Borel parameter M^2 as independent equations with estimated errors. Then we find the minimal χ^2 solution of the system of such equations and test its stability against variations of different parameters involved. In section 2 we formulate the problem and present the equations used for condensate determination. In section 3 we describe the procedure for solution of a system of sum rules. Sect.4 contains the presentation of results, discussion and comments.

Equations

In order to impose the bounds on the possible values of different condensates and δ^2 we consider the following set of two point functions of the hybrid current a_μ :

$$\begin{aligned}
i \int dx e^{iqx} < 0 | T \left(\bar{u} \gamma_\alpha \tilde{G}_{\alpha\mu} d(x), \bar{d} \gamma_\beta \tilde{G}_{\beta\nu} u(0) \right) | 0 > &= g_{\mu\nu} \Pi_1^d(q^2) + q_\mu q_\nu \Pi_2^d(q^2) \\
i \int dx e^{iqx} < 0 | T \left(\bar{u} \gamma_\mu \gamma_5 d(x), \bar{d} \gamma_\rho \tilde{G}_{\rho\nu} u(0) \right) | 0 > &= g_{\mu\nu} \Pi_1^{ax}(q^2) + q_\mu q_\nu \Pi_2^{ax}(q^2) \quad (4) \\
i \int dx e^{iqx} < 0 | T \left(\bar{u} \gamma_5 d(x), \bar{d} \gamma_\rho \tilde{G}_{\rho\mu} u(0) \right) | 0 > &= q_\mu \Pi^{ps}(q^2)
\end{aligned}$$

There are several reasons which make this correlators appropriate for condensate determination. First of all, their perturbative expansions start at the two loop level, so the nonperturbative terms become dominant and hence easier to determine. In contrast to the vector and axial-vector current correlators, in the operator expansion (OPE) of the two point functions (4) the correlation of gluon and four-quark condensates values is negative, which allows to bound their absolute values from above. The number of unknown parameters in the sum rules is reduced by the fact that imaginary parts of this correlators are contributed by the same physical states. These are the pion, a_1 and higher mass states which in the following will be attributed to the continuum. In the sum rules we use the usual model spectrum with π and a_1 as narrow resonances and the continuum equal to the perturbative one, starting at some point s_0 . After standard manipulations one obtains the following SRs for $\Pi_2^d, \Pi_2^{ax} \Pi_1^{ax}$ and Π^{ps} correspondingly:

$$\begin{aligned}
\frac{1}{72} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{st} G + \frac{8}{9} \frac{\langle \sqrt{\alpha_s} \bar{q} q \rangle_{fac}^2 Q}{M^2} &= \\
= \frac{f_\pi^2 \delta^4}{M^2} + \frac{f_a^2 \delta_a^4}{M^2} \exp(-m_a^2/M^2) - \frac{\alpha_s}{80\pi^3} M^4 \left(1 - (1 + x_0 + \frac{x_0^2}{2}) e^{-x_0} \right) &\quad (5)
\end{aligned}$$

$$\begin{aligned}
\frac{1}{6M^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{st} G + \frac{64\pi}{27} \frac{\langle \sqrt{\alpha_s} \bar{q} q \rangle_{fac}^2 Q}{M^4} &= \\
= \frac{f_\pi^2 \delta^2}{M^2} - \frac{f_a^2 \delta_a^2}{M^2} \exp(-m_a^2/M^2) - \frac{\alpha_s}{18\pi^3} M^2 \left(1 - (1 + x_0) e^{-x_0} \right) &\quad (6)
\end{aligned}$$

$$\begin{aligned}
\frac{64\pi}{27} \frac{\langle \sqrt{\alpha_s} \bar{q} q \rangle_{fac}^2 Q}{M^2} &= \\
= \frac{f_a^2 m_a^2 \delta_a^2}{M^2} \exp(-m_a^2/M^2) - \frac{\alpha_s}{18\pi^3} M^4 \left(1 - (1 + x_0 + \frac{x_0^2}{2}) e^{-x_0} \right) &\quad (7)
\end{aligned}$$

$$\frac{m_0^2 \langle \bar{q} q \rangle}{M^2} + \frac{\pi^2}{9} \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle \langle \bar{q} q \rangle}{M^4} = \frac{\delta^2 \langle \bar{q} q \rangle}{M^2} - \frac{\alpha_s \langle \bar{q} q \rangle}{3\pi} (1 - e^{-x'_0}) \quad (8)$$

where $f_\pi = 133$ MeV and $f_a = 170$ MeV[1] are the leptonic decay constants of π and a_1 mesons correspondingly. $m_a = 1260$ MeV is the a_1 mass and in an analogy with the pion matrix element we have $\epsilon_\mu f_a \delta_a^2 = \langle 0 | a_\mu | a_1 \rangle$. We have introduced the coefficients

$$G \equiv \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle / \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{st} \quad Q \equiv \langle (\bar{q}\Gamma q)^2 \rangle / \langle (\bar{q}\Gamma q)^2 \rangle_{fac}$$

measuring the deviations of corresponding condensates from their standard values. Note that Q is some average coefficient for different four-quark condensates which appear in the operator product expansion (OPE) of the considered correlators. We have $x_0 = \frac{s_0}{M^2}$ where s_0 is the continuum onset in eqs.(5-7). On the other hand in eq.(8) where there is no a_1 contribution the corresponding continuum threshold is taken to be lower $s'_0 < s_0$. We disregard the anomalous dimensions of the operators and consider the corresponding matrix elements at $\mu^2 \approx M^2$. All the terms except the unknown condensate contributions are moved to the right hand sides of the corresponding equations.

A few comments are in order before we proceed further. Equations (5) and (6) are taken from [7]. In eq.(7) we have corrected the sign of the second term in the rhs. which was incorrect in ref.[8]. This change reduces the resulting value of quark-gluon condensate considerably. The SR (8) is not very reliable since its different sides show different M^2 behavior, however we have added it to the system just to illustrate some points of our approach. The second term of the lhs. of (7) is obtained by the factorization of dimension 7 quark-gluon condensate, however due to numerical smallness of this term the approximation has practically no effect on the results.

Now let us recall the standard sum rules procedure in order to understand better our proposal. Note that QCD sum rules are approximate equations which depend on the borel parameter M^2 . The errors in this equations are mainly caused by two independent reasons. At low M^2 the inaccuracy of equations is mostly caused by the errors resulting from the truncation of OPE series and from the uncertainties of theoretical parameters. Whereas at high values of borel parameter the errors caused by inaccuracies of experimental (or model) spectra become more essential. At the intermediate values of the Borel parameter there is some interplay of both of these errors. If there is a region of M^2 where both of them are moderate ("fiducial region"), then the physical quantity in question is extracted by equating theoretical and experimental parts of SR, as well as their derivatives, at some particular point within this region. The reliability of the result is then tested by its stability against variation of M^2 .

The above procedure is sufficient for deriving qualitative results and estimates, however it contains considerable amount of arbitrariness related to the choice of fiducial region, determination of the central value and errors of the final result. Besides it is difficult to compare the results obtained in different sum rules by means of such a procedure. The well established quantitative results can be obtained by QCD sum rules method only in case if these uncertainties will be somehow eliminated.

Instead of the traditional treatment we suggest to consider the Borel transformed SR (5-8) at different M^2 as independent approximate equations in which the esti-

mates of the expected errors and their correlations should be done. Then the system of such equations can be solved in the sense of minimal χ^2 , with account of this errors. This provides a more natural concept of similarity of different parts of the sum rules than the standard procedure. Calculating χ^2 per degree of freedom we have the measure of consistency of different sum rules entering the system. Then we scan all the possible values of the unknown parameters δ^2 and δ_a^2 and find the region in the three dimensional space of parameters G , Q and m_0 for which the system is consistent. This gives us the bounds on the possible values of condensates as well as correlations among them.

Of course such a way is related to many subtleties of the error analysis, however this is unavoidable if one wants to controll the precision of the results obtained by the sum rules method. At least moving in this direction we will investigate the capability of the method to provide accurate numerical results.

Condensate determination

In order to determine the allowed ranges for the gluon, four-quark and quark-gluon condensates we perform the following steps:

1. Construction of the system of equations. Taking some values of the parameters δ^2 and δ_a^2 we evaluate the equations (5-8) at four different values of the borel parameter M_α^2 $\alpha = 1, ., 4$ and build up a system of 16 linear equations for determination of the parameters Q , G and m_0^2 . We take the M_α^2 points in the range $0.6 \div 2 \text{ GeV}^2$, however they should not be too close to each other in order to avoid strong error correlations among the resulting equations (see below).

2. The estimate of expected errors. To set the scale of expected errors in the resulting equations we first check separately the SRs (5-8) and adopt some relative errors w_i for each of them at $M^2 = 1 \text{ GeV}^2$. The eqs. (5-7) are easily stabilized by appropriate choice of s_0 for reasonable values of condensates and δ^2 . Hence, we have no reason to expect that some important contributions are missing. On the other hand eq.(8) exhibits different M^2 behavior of its right and left hand sides and is clearly less reliable. So, we should take $w_8 > w_{5,6,7}$. As initial guess we assume $w_{5,6,7} = 0.1$ and $w_8 = 0.3$ i.e. 10% relative error at $M^2 = 1 \text{ GeV}^2$ for eqs.(5-7) and 30% of that for (8). We require that the results should be stable against reasonable variations of these numbers.

The M^2 dependence of errors is taken to be common for all the considered SRs and is described by the error distribution function

$$D(M^2) = \begin{cases} \frac{1-M_0^2}{M^2-M_0^2} & \text{if } M^2 < 1 \text{ GeV}^2 \\ 1 & \text{if } M^2 \geq 1 \text{ GeV}^2 \end{cases} \quad (9)$$

Such a choice of $D(M^2)$ can be justified in the following way; It is known that at low M^2 the higher order power corrections in the OPE series become essential and at some M^2 the expansion blows up. So, we assume the pole like behaviour at $M^2 < 1 \text{ GeV}^2$ with M_0^2 being some effective convergence radius of the series. On the

other hand for the higher M^2 values the contributions of a_1 and higher states, which become the main source of the errors, remain practically constant, so we assume that the error distribution is uniform for $M^2 > 1 \text{ GeV}^2$. Note that the absolute values of SRs decrease with M^2 so the above of $D(M^2)$ corresponds to increasing *relative* error both for high and low M^2 .

Finally, the expected *absolute* errors for the equations of our system are calculated as

$$\Delta_i(M_\alpha^2) = \omega_i D(M_\alpha^2) A_i(M_\alpha^2) \quad (10)$$

where $A_i(M^2)$ denotes the rhs.s of (5-8).

3.Solution of the system of equations. To find the allowed regions of the condensate values we take different values of the parameters δ^2 and δ_a^2 and for every combination we find the solution for Q, G and m_0 at which the system is maximally consistent. Such a way is dictated by simplicity reasons, since the system is linear in condensates. As a measure of consistency of the system we use the minimal χ^2 criterium the use of which we will now try to advocate.

It is clear, that the errors of our equations are not statistical and should be somehow correlated with each other. However these correlations depend on many different factors, like unknown higher order terms in OPE, perturbative corrections, discrepancies between model and real spectra, possible non OPE contributions etc. The knowledge of all these factors would be almost equivalent to the solution of QCD for our case, and we are clearly far from this. The best that we can do at present, is to consider the errors of different SRs as uncorrelated; As for errors within each of the sum rules (5-8), we have to keep the M^2 points sufficiently separated to ensure that this correlations are small. The effect of our assumption will be less important for a greater number of equations, and can be partly compensated by increasing the error bars. In other words we suggest to treat the uncertainties caused by a great number of unknown parameters as some statistical errors. So, assuming that in each of 16 equations the errors are independent we sum them quadratically to calculate χ^2 per degree of freedom.

We scan all the possible values of the parameters δ^2 and δ_a^2 and for each pair find the corresponding values of G, Q and m_0^2 with minimal $\chi_{d.f.}^2$. If the resulting $\chi_{min}^2 < 1$ then the system is considered to be consistent and the values of the above parameters - allowed. Thus we obtain the allowed region in the three dimensional space of parameters G, Q and m_0^2 . The corresponding region in the plain of (G, Q) is shown in fig.1. The inner curve - *a*) corresponds to $\chi_{df.min}^2 = 1$ However to be less stringent we allow also solutions with $\chi_{df.min}^2 \leq 2$ (bounded by the curve *b*)) which corresponds to $\approx 15\%$ relative error at $M^2 = 1 \text{ GeV}^2$ in eqs.(5-7) .

One should keep in mind that for each δ^2 and δ_a^2 the parameters G, Q and m_0^2 , giving minimal χ_{df}^2 solution, have correlated errors which have also to be taken into account. For a few solutions this is illustrated by ellipses in fig.2 . These ellipses, for every solution with χ_{min}^2 , include the points for which $\chi^2 \leq \chi_{min}^2 + 1$. The area covered by all these ellipses is similar to the area with $\chi_{min}^2 \leq 2$ bounded by the curve *b*) of fig.1.

4.Stability check and parameter adjustment. The results should be stable against the variations of the parameters involved in the derivation. So we check the stability of the boundaries and individual solutions, and adjust the parameters in order to obtain the maximal stability of the allowed region.

If the error distribution function is realistic, then the results should not depend considerably on the variation of the endpoints M_1^2 and M_4^2 . The error distribution function (9) with $M_0^2 = 0.45 \text{ GeV}^2$ just satisfies this requirement. The boundary of the allowed region remains practically unchanged when M_1^2 and M_4^2 vary in the intervals $(0.6 - 0.9) \text{ GeV}^2$ and $(1.6 - 2) \text{ GeV}^2$ respectively. The region shown in fig.1 is obtained with M_α^2 values taken at 0.7,1.1, 1.4,1.9 GeV^2 . For comparison in fig.3 we have also plotted dependence of the allowed (G, Q) region on the lower endpoint M_1^2 for $M_0^2 = 0.3 \text{ GeV}^2$.

The final result is also insensitive to individual variations of w_i within about a factor of 2. However, this is not so for w_8 and the allowed region feels the erroneous eq.(8) unless we take $w_8 \geq 0.5$. Our final result is not strongly affected by the considerable variations of s_0 and s'_0 as can be seen from fig.4. This however is hardly surprising in view of the two loop suppression of perturbative contributions.

Results and discussion

Thus, we have found the allowed regions for the gluon, four-quark and quark-gluon condensates as well as the parameters δ^2 and δ_A^2 . We conclude that the system of sum rules (5-8) is consistent with the standard values of condensates. From fig.s 1,2 one can see that the gluon condensate can differ from its SV at most by a factor of 2 and this happens if the four-quark condensates are close to their factorized values. On the other hand the latter are less tied to their SV and can exceed them 3-4 times. The upper bounds on the values of condensates are more strict than that derived from vector and axial channels. This is due to the fact that in the considered equations both the gluon and the four-quark condensates have the same sign and thus cannot compensate the growth of each other. Note however that we did not distinguish different types of the four-quark condensates.

Since for every δ and δ_a^2 we obtain a triplet of parameters G , Q and m_0^2 , we thus have some functional dependence among their possible values, described by two dimensional surface in the three-dimensional space of this parameters. This dependence is almost linear and can be expressed by the formula:

$$m_0^2 = (0.24G + 0.3Q + 0.09) \text{ GeV}^2$$

which for the SV of condensates (i.e. $G=Q=1$) yields:

$$m_0 = 0.63 \text{ GeV}^2$$

in difference with eq.(3)

The allowed regions of condensates correspond to the following ranges of the pion and a_1 matrix elements: $0.1 \text{ MeV}^2 < \delta^2 < 0.3 \text{ MeV}^2$ and $0 < \delta_a^2 < 0.2 \text{ MeV}^2$ which are thus also bounded by the requirement of consistency of the system of SR.

The proposed procedure of analysis can be easily extended to other systems of two point functions and to a greater number of equations, however in our opinion it is already useful for analysis of a single SR, since it allows clear and testable assumptions about errors and their distribution. We hope, that this will be helpful for further restriction of condensate values or establishing the limits of applicability of the QCD SR method.

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Figure captions

Fig.1 The allowed region in the plane (G, Q) calculated for $M_i^2 = 0.7, 1.1, 1.4, 1.9 \text{ GeV}^2$. The curve **a)** bounds the region with $\chi_{min}^2 \leq 1$ assuming 10% rel.error in eqs.(5-7) at $M^2 = 1 \text{ GeV}^2$. **b)** bounds the region with $\chi_{min}^2 \leq 2$.

Fig.2 The allowed region of (G, Q) plane taking into account the errors of the solutions with $\chi_{min}^2 \leq 1$ (inside curve a) of fig.1). The ellipses indicate the areas with $\chi^2 \leq \chi_{min}^2 + 1$ for three different solutions, one of them corresponding to the SVs.

Fig.3 Allowed (G, Q) regions with $\chi_{min}^2 \leq 1$ calculated using the error distribution function (9) with $M_0^2 = 0.3 \text{ GeV}^2$; a)for $M_1^2 = 0.6 \text{ GeV}^2$ b)for $M_1^2 = 0.8 \text{ GeV}^2$.

Fig.4 Allowed regions with $\chi_{min}^2 \leq 1$ for different choice of s_0 and s'_0 .
a) - $s_0 = 2 \text{ GeV}^2$ $s'_0 = 1.5 \text{ GeV}^2$ b) - $s_0 = 3 \text{ GeV}^2$ $s'_0 = 2.5 \text{ GeV}^2$